OBLIQUE SHOCK WITH TOTAL CONDENSATION

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The present paper examines the problem of an oblique condensation shock under the assumption that the condensation coefficient is unity [1].

Such a formulation of the problem follows, for example, from an examination of the physical processes on a surface element of the cavity formed when a vapor discharges into a volume filled with relatively cold liquid. It is assumed that the heat of condensation is transferred from the vapor to the liquid and is completely removed from the condensation boundary by the liquid stream. Thus, the concept of a "condensation shock" refers to the comparatively wide layer in which turbulent release of heat to the liquid is accomplished.

The general theory of cavities on whose surfaces phase transition takes place is far from complete, especially as regards turbulent flow. A semi-empirical theory based on the work of Glikman [2] permits determination both of the location of the "condensation front" and the parameters of the turbulent boundary layer behind the "condensation front." For a preliminary estimate, however, we may restrict our attention to the one-dimensional case, and, by dividing the cavity surface into a number of plane sections, we may calculate the liquid flow parameters and heat transfer.

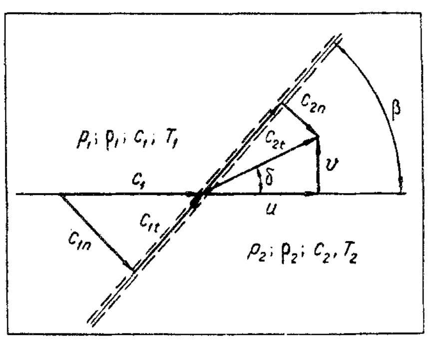

Fig. 1. Oblique condensation shock.

We shall examine the following problem. A homogeneous stream of vapor falls at an angle on a plane liquid surface (Fig. 1).

The laws of conservation relating the flow parameters ahead of and behind the shock may be written in the form

$$\rho_1 c_{1n} = \rho_2 c_{2n}, \tag{1}$$

$$p_1 + \rho_1 c_{1n}^2 = p_2 + \rho_2 c_{2n}^2, \tag{2}$$

$$c_{1t} = c_{2t}, \tag{3}$$

$$\frac{c_1^2}{2} + i_1 = \frac{c_2^2}{2} + i_2 + q. \tag{4}$$

The subscript 1 refers to quantities describing the flow ahead of the shock, and the subscript 2 to quantities behind the shock, which are to be determined (with the exception of ρ_2 and i_2).

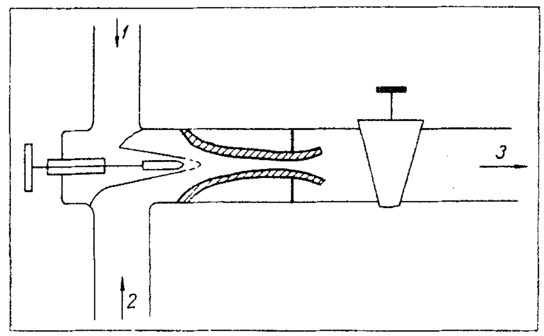

Fig. 2. Schematic of a vapor-liquid injector: 1) vapor; 2) liquid; 3) total flow.

The system of equations (1)–(4) remains valid for any equation of state of the working substance, both before and after the condensation shock. In the given case we shall assume that the working substance ahead of the shock is a gas, and behind the shock—an incompressible liquid. In this situation the density discontinuity is defined independently of the remaining parameters, and the relative density $\gamma = \rho_1/\rho_2$ is a parameter of the problem.

Designating the velocity component behind the shock parallel to the original flow direction by u, and the component perpendicular to it by v (see Fig. 1), we may write (3) in the form

$$c_1 \cos\beta = u \cos\beta + v \sin\beta. \tag{5}$$

Noting that

$$c_{1n} = c_1 \sin\beta,$$
$$c_{2n} = u \sin\beta - v \cos\beta,$$

we write (1) in the form

$$c_1 \cdot \gamma = u - v \cdot \operatorname{ctg}\beta. \tag{6}$$

Solving (5) and (6) simultaneously, we find

$$u = c_1 [1 - (1 - \gamma) \sin^2\beta], \tag{7}$$

$$v = \frac{c_1(1-\gamma)}{2} \sin 2\beta, \tag{8}$$

and for the total velocity of the liquid

$$c_2 = c_1 \sqrt{1 - (1 - \gamma^2) \sin^2\beta}. \tag{9}$$

The angle of inclination of the flow behind the shock is determined by the relation

$$\operatorname{tg}\delta = v/u = (1-\gamma)\sin 2\beta / 2\{1 - (1-\gamma)\sin^2\beta\}. \tag{10}$$

Taking account of (2) and the expressions for c_{1n} and c_{2n}, the relation between the pressures before and after the shock may be written in the form

$$p_2 = p_1 + \rho_1 c_1^2 (1-\gamma)\sin^2\beta. \qquad (11)$$

For the stagnation pressures p_{01} and p_{02} the relation takes the form

$$p_{02} = p_{01} + \frac{\rho_1 c_1^2}{2} \cdot \frac{1-\gamma}{\gamma} \cdot \left[1-(1-\gamma)\sin^2\beta\right]. \qquad (12)$$

Here the amount of heat that must be removed from the condensation boundary to attain a steady state is determined from (4) to be

$$q = \Delta i + \frac{c_1^2}{2} (1-\gamma)\sin^2\beta, \qquad (13)$$

where $\Delta i = i_1 - i_2$.

Thus, the stagnation pressure in the liquid stream is always greater than in the vapor stream, if condensation occurs at the shock, and the latent heat of vaporization is removed from the liquid.

This effect (or something close to it) is evidently used in vapor-liquid injectors. This type of injector (Fig. 2) was customarily used for admitting water into steam boilers [3]. It has been pointed out in the literature, however, that such injectors could find wider application in special-purpose equipment, because of their simplicity and reliability of construction.

Thus, for instance, the possibility is examined in [4, 5] of using jet pumps with vapor-liquid injectors to supply fuel to liquid rocket motors.

NOTATION

c (m/sec)—total velocity; c_n and c_t (m/sec)—normal and tangential velocity components; u (m/sec)—component of vapor velocity parallel to liquid velocity; v (m/sec)—component of vapor velocity perpendicular to liquid velocity; ρ (kg/m^3)—density; p (N/m^2)—static pressure p_0 (N/m^2)—stagnation pressure; i (j/kg)—enthalpy per unit mass; q (j/kg)—amount of heat removed from unit mass of liquid; γ—ratio of vapor density to liquid density; β—angle between direction of stream ahead of condensation shock and condensation surface; δ—angle through which flow is deviated from its original direction. Subscript 1 refers to quantities describing the vapor phase, and subscript 2—to quantities describing the liquid phase.

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